

TEXES
Texas Examinations of Educator Standards

Field 115: Mathematics 4–8
Test Framework

Domain	Range of Competencies
I. Number Concepts	001–003
II. Patterns and Algebra	004–007
III. Geometry and Measurement	008–011
IV. Probability and Statistics	012–014
V. Mathematical Processes and Perspectives	015–016
VI. Mathematical Learning, Instruction, and Assessment	017–019

TEST FRAMEWORK FOR FIELD 115: MATHEMATICS 4–8

Domain I Number Concepts (approximately 16% of the test)

Standards Assessed:

Mathematics Standard I:

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Domain II Patterns and Algebra (approximately 21% of the test)

Standards Assessed:

Mathematics Standard II:

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Domain III Geometry and Measurement (approximately 21% of the test)

Standards Assessed:

Mathematics Standard III:

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Domain IV Probability and Statistics (approximately 16% of the test)

Standards Assessed:

Mathematics Standard IV:

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Domain V Mathematical Processes and Perspectives (approximately 10% of the test)

Standards Assessed:

Mathematics Standards V and VI:

Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.

Domain VI Mathematical Learning, Instruction, and Assessment (approximately 16% of the test)

Standards Assessed:

Mathematics Standards VII and VIII:

Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures, and concepts, knows typical errors students make, and uses this knowledge to plan, organize, and implement instruction; to meet curriculum goals; and to teach all students to understand and use mathematics.

Mathematical Assessment: The mathematics teacher understands assessment and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

TEXES
Texas Examinations of Educator Standards

Field 115: Mathematics 4–8
Test Framework

DOMAIN I—NUMBER CONCEPTS

Competency 001

The teacher understands the structure of number systems, the development of a sense of quantity, and the relationship between quantity and symbolic representations.

The beginning teacher:

- Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- Understands the relative magnitude of whole numbers, integers, rational numbers, and real numbers.
- Demonstrates an understanding of a variety of models for representing numbers (e.g., fraction strips, diagrams, patterns, shaded regions, number lines).
- Demonstrates an understanding of equivalency among different representations of rational numbers.
- Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Understands the characteristics of the set of whole numbers, integers, rational numbers, real numbers, and complex numbers (e.g., commutativity, order, closure, identity elements, inverse elements, density).
- Demonstrates an understanding of how some situations that have no solution in one number system (e.g., whole numbers, integers, rational numbers) have solutions in another number system (e.g., real numbers, complex numbers).

Competency 002

The teacher understands number operations and computational algorithms.

The beginning teacher:

- Works proficiently with real and complex numbers and their operations.
- Analyzes and describes relationships between number properties, operations, and algorithms for the four basic operations involving integers, rational numbers, and real numbers.

FIELD 115: MATHEMATICS 4–8 TEST FRAMEWORK

- Uses a variety of concrete and visual representations to demonstrate the connections between operations and algorithms.
- Justifies procedures used in algorithms for the four basic operations with integers, rational numbers, and real numbers, and analyzes error patterns that may occur in their application.
- Relates operations and algorithms involving numbers to algebraic procedures (e.g., adding fractions to adding rational expressions, division of integers to division of polynomials).
- Extends and generalizes the operations on rationals and integers to include exponents, their properties, and their applications to the real numbers.

Competency 003

The teacher understands ideas of number theory and uses numbers to model and solve problems within and outside of mathematics.

The beginning teacher:

- Demonstrates an understanding of ideas from number theory (e.g., prime factorization, greatest common divisor) as they apply to whole numbers, integers, and rational numbers, and uses these ideas in problem situations.
- Uses integers, rational numbers, and real numbers to describe and quantify phenomena such as money, length, area, volume, and density.
- Applies knowledge of place value and other number properties to develop techniques of mental mathematics and computational estimation.
- Applies knowledge of counting techniques such as permutations and combinations to quantify situations and solve problems.
- Applies properties of the real numbers to solve a variety of theoretical and applied problems.

DOMAIN II—PATTERNS AND ALGEBRA

Competency 004

The teacher understands and uses mathematical reasoning to identify, extend, and analyze patterns and understands the relationships among variables, expressions, equations, inequalities, relations, and functions.

The beginning teacher:

- Uses inductive reasoning to identify, extend, and create patterns using concrete models, figures, numbers, and algebraic expressions.
- Formulates implicit and explicit rules to describe and construct sequences verbally, numerically, graphically, and symbolically.

**FIELD 115: MATHEMATICS 4–8
TEST FRAMEWORK**

- Makes, tests, validates, and uses conjectures about patterns and relationships in data presented in tables, sequences, or graphs.
- Gives appropriate justification of the manipulation of algebraic expressions.
- Illustrates the concept of a function using concrete models, tables, graphs, and symbolic and verbal representations.
- Uses transformations to illustrate properties of functions and relations and to solve problems.

Competency 005

The teacher understands and uses linear functions to model and solve problems.

The beginning teacher:

- Demonstrates an understanding of the concept of linear function using concrete models, tables, graphs, and symbolic and verbal representations.
- Demonstrates an understanding of the connections among linear functions, proportions, and direct variation.
- Determines the linear function that best models a set of data.
- Analyzes the relationship between a linear equation and its graph.
- Uses linear functions, inequalities, and systems to model problems.
- Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of linear equations and inequalities.
- Demonstrates an understanding of the characteristics of linear models and the advantages and disadvantages of using a linear model in a given situation.

Competency 006

The teacher understands and uses nonlinear functions and relations to model and solve problems.

The beginning teacher:

- Uses a variety of methods to investigate the roots (real and complex), vertex, and symmetry of a quadratic function or relation.
- Demonstrates an understanding of the connections among geometric, graphic, numeric, and symbolic representations of quadratic functions.
- Analyzes data and represents and solves problems involving exponential growth and decay.
- Demonstrates an understanding of the connections among proportions, inverse variation, and rational functions.
- Understands the effects of transformations such as $f(x \pm c)$ on the graph of a nonlinear function $f(x)$.

FIELD 115: MATHEMATICS 4–8 TEST FRAMEWORK

- Applies properties, graphs, and applications of nonlinear functions to analyze, model, and solve problems.
- Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of quadratic equations and inequalities.
- Understands how to use properties, graphs, and applications of non-linear relations including polynomial, rational, radical, absolute value, exponential, logarithmic, trigonometric, and piecewise functions and relations to analyze, model, and solve problems.

Competency 007

The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics.

The beginning teacher:

- Relates topics in middle school mathematics to the concept of limit in sequences and series.
- Relates the concept of average rate of change to the slope of the secant line and instantaneous rate of change to the slope of the tangent line.
- Relates topics in middle school mathematics to the area under a curve.
- Demonstrates an understanding of the use of calculus concepts to answer questions about rates of change, areas, volumes, and properties of functions and their graphs.

DOMAIN III—GEOMETRY AND MEASUREMENT

Competency 008

The teacher understands measurement as a process.

The beginning teacher:

- Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.
- Develops, justifies, and uses conversions within measurement systems.
- Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- Describes the precision of measurement and the effects of error on measurement.
- Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.

**FIELD 115: MATHEMATICS 4–8
TEST FRAMEWORK**

Competency 009

The teacher understands the geometric relationships and axiomatic structure of Euclidean geometry.

The beginning teacher:

- Understands concepts and properties of points, lines, planes, angles, lengths, and distances.
- Analyzes and applies the properties of parallel and perpendicular lines.
- Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
- Describes and justifies geometric constructions made using a compass and straight edge and other appropriate technologies.
- Applies knowledge of the axiomatic structure of Euclidean geometry to justify and prove theorems.

Competency 010

The teacher analyzes the properties of two- and three-dimensional figures.

The beginning teacher:

- Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
- Applies relationships among similar figures, scale, and proportion and analyzes how changes in scale affect area and volume measurements.
- Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two- and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.
- Analyzes the relationship among three-dimensional figures and related two-dimensional representations (e.g., projections, cross-sections, nets) and uses these representations to solve problems.

Competency 011

The teacher understands transformational geometry and relates algebra to geometry and trigonometry using the Cartesian coordinate system.

The beginning teacher:

- Describes and justifies geometric constructions made using a reflection device and other appropriate technologies.
- Uses translations, reflections, glide-reflections, and rotations to demonstrate congruence and to explore the symmetries of figures.
- Uses dilations (expansions and contractions) to illustrate similar figures and proportionality.
- Uses symmetry to describe tessellations and shows how they can be used to illustrate geometric concepts, properties, and relationships.

**FIELD 115: MATHEMATICS 4–8
TEST FRAMEWORK**

- Applies concepts and properties of slope, midpoint, parallelism, and distance in the coordinate plane to explore properties of geometric figures and solve problems.
- Applies transformations in the coordinate plane.
- Uses the unit circle in the coordinate plane to explore properties of trigonometric functions.

DOMAIN IV—PROBABILITY AND STATISTICS

Competency 012

The teacher understands how to use graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns.

The beginning teacher:

- Organizes and displays data in a variety of formats (e.g., tables, frequency distributions, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- Applies concepts of center, spread, shape, and skewness to describe a data distribution.
- Supports arguments, makes predictions, and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.
- Demonstrates an understanding of measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, interquartile range, variance, standard deviation).
- Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers, and measures of central tendency and dispersion.
- Calculates and interprets percentiles and quartiles.

Competency 013

The teacher understands the theory of probability.

The beginning teacher:

- Explores concepts of probability through data collection, experiments, and simulations.
- Uses the concepts and principles of probability to describe the outcome of simple and compound events.
- Generates, simulates, and uses probability models to represent a situation.

**FIELD 115: MATHEMATICS 4–8
TEST FRAMEWORK**

- Determines probabilities by constructing sample spaces to model situations.
- Solves a variety of probability problems using combinations, permutations, and geometric probability (i.e., probability as the ratio of two areas).
- Uses the binomial, geometric, and normal distributions to solve problems.

Competency 014

The teacher understands the relationship among probability theory, sampling and statistical inference, and how statistical inference is used in making and evaluating predictions.

The beginning teacher:

- Applies knowledge of designing, conducting, analyzing, and interpreting statistical experiments to investigate real-world problems.
- Demonstrates an understanding of random samples, sample statistics, and the relationship between sample size and confidence intervals.
- Applies knowledge of the use of probability to make observations and draw conclusions from single variable data and to describe the level of confidence in the conclusion.
- Makes inferences about a population using binomial, normal, and geometric distributions.
- Demonstrates an understanding of the use of techniques such as scatter plots, regression lines, correlation coefficients, and residual analysis to explore bivariate data and to make and evaluate predictions.

DOMAIN V—MATHEMATICAL PROCESSES AND PERSPECTIVES

Competency 015

The teacher understands mathematical reasoning and problem solving.

The beginning teacher:

- Demonstrates an understanding of proof, including indirect proof, in mathematics.
- Applies correct mathematical reasoning to derive valid conclusions from a set of premises.
- Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.
- Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.

**FIELD 115: MATHEMATICS 4–8
TEST FRAMEWORK**

- Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
- Evaluates the reasonableness of a solution to a given problem.
- Applies content knowledge to develop a mathematical model of a real-world situation and analyzes and evaluates how well the model represents the situation.
- Demonstrates an understanding of estimation and evaluates its appropriate uses.

Competency 016

The teacher understands mathematical connections within and outside of mathematics and how to communicate mathematical ideas and concepts.

The beginning teacher:

- Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of circle as a quadratic function in r , probability as the ratio of two areas).
- Uses mathematics to model and solve problems in other disciplines, such as art, music, science, social science, and business.
- Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
- Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
- Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
- Uses the language of mathematics as a precise means of expressing mathematical ideas.
- Understands the structural properties common to the mathematical disciplines.

DOMAIN VI—MATHEMATICAL LEARNING, INSTRUCTION, AND ASSESSMENT

Competency 017

The teacher understands how children learn and develop mathematical skills, procedures, and concepts.

The beginning teacher:

- Applies theories and principles of learning mathematics to plan appropriate instructional activities for all students.
- Understands how students differ in their approaches to learning mathematics with regards to diversity.

FIELD 115: MATHEMATICS 4–8 TEST FRAMEWORK

- Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- Understands how learning may be assisted through the use of mathematics manipulatives and technological tools.
- Understands how to motivate students and actively engage them in the learning process by using a variety of interesting, challenging, and worthwhile mathematical tasks in individual, small-group, and large-group settings.
- Understands how to provide instruction along a continuum from concrete to abstract.
- Recognizes the implications of current trends and research in mathematics and mathematics education.

Competency 018

The teacher understands how to plan, organize, and implement instruction using knowledge of students, subject matter, and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to teach all students to use mathematics.

The beginning teacher:

- Demonstrates an understanding of a variety of instructional methods, tools, and tasks that promote students' ability to do mathematics described in the TEKS.
- Understands planning strategies for developing mathematical instruction as a discipline of interconnected concepts and procedures.
- Develops clear learning goals to plan, deliver, assess, and reevaluate instruction based on the TEKS.
- Understands procedures for developing instruction that establishes transitions between concrete, symbolic, and abstract representations of mathematical knowledge.
- Applies knowledge of a variety of instructional delivery methods, such as individual, structured small-group, and large-group formats.
- Understands how to create a learning environment that provides all students, including English Language Learners, with opportunities to develop and improve mathematical skills and procedures.
- Demonstrates an understanding of a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.
- Understands how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending, and applying mathematical concepts.
- Understands how to relate mathematics to students' lives and a variety of careers and professions.

**FIELD 115: MATHEMATICS 4–8
TEST FRAMEWORK**

Competency 019

The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

The beginning teacher:

- Demonstrates an understanding of the purpose, characteristics, and uses of various assessments in mathematics, including formative and summative assessments.
- Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- Demonstrates an understanding of how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns.
- Understands how to evaluate a variety of assessment methods and materials for reliability, validity, absence of bias, clarity of language, and appropriateness of mathematical level.
- Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor, and modify instruction to improve mathematical learning for all students, including English Language Learners.